A Sacks amoeba preserving distributivity of $\mathcal{P}(\omega)/fin$

Wolfgang Wohofsky

joint work with Otmar Spinas

till today: Universität Kiel, Germany

from tomorrow: Kurt Gödel Research Center, Vienna (KGRC)

wolfgang.wohofsky@gmx.at

Winter School in Abstract Analysis, section Set Theory
Hejnice, Czech Republic, 26th Jan – 2th Feb 2019

Main Theorem (Spinas-W.)

Assume CH. Let $\langle(\mathbb{P}_\alpha, \dot{\mathbb{Q}}_\alpha) : \alpha < \omega_2\rangle$ be the countable support iteration with $\Vdash_{\mathbb{P}_\alpha} \dot{\mathbb{Q}}_\alpha = \dot{\mathbb{A}}$ for each $\alpha < \omega_2$.
Then in the final model $V^{\mathbb{P}_{\omega_2}}$ we have

$$\omega_1 = \mathrm{cov}(\mathcal{N}) = \mathrm{cov}(\mathcal{M}) = \mathfrak{h} < \mathrm{add}(s^0) = \mathfrak{s} = \mathfrak{b} = \mathfrak{c} = \omega_2.$$

Main Theorem (Spinas-W.)

Assume CH. Let $\langle(\mathbb{P}_\alpha, \dot{\mathbb{Q}}_\alpha) : \alpha < \omega_2\rangle$ be the countable support iteration with $\Vdash_{\mathbb{P}_\alpha} \dot{\mathbb{Q}}_\alpha = \dot{\mathbb{A}}$ for each $\alpha < \omega_2$.
Then in the final model $V^{\mathbb{P}_{\omega_2}}$ we have

$$\omega_1 = \mathrm{cov}(\mathcal{N}) = \mathrm{cov}(\mathcal{M}) = \mathfrak{h} < \mathrm{add}(s^0) = \mathfrak{s} = \mathfrak{b} = \mathfrak{c} = \omega_2.$$

Main Theorem (Spinas-W.)

Assume CH. Let $\langle(\mathbb{P}_\alpha, \dot{\mathbb{Q}}_\alpha) : \alpha < \omega_2\rangle$ be the countable support iteration with $\Vdash_{\mathbb{P}_\alpha} \dot{\mathbb{Q}}_\alpha = \dot{\mathbb{A}}$ for each $\alpha < \omega_2$.
Then in the final model $V^{\mathbb{P}_{\omega_2}}$ we have

$$\omega_1 = \mathrm{cov}(\mathcal{N}) = \mathrm{cov}(\mathcal{M}) = \mathfrak{h} < \mathrm{add}(s^0) = \mathfrak{s} = \mathfrak{b} = \mathfrak{c} = \omega_2.$$

$$\omega_1 = \mathrm{cov}(\mathcal{N}) = \mathrm{cov}(\mathcal{M}) = \mathfrak{h} < \mathrm{add}(s^0) = \mathfrak{s} = \mathfrak{b} = \mathfrak{c} = \omega_2.$$

Definition

Let $\mathbb{P}$ be a tree forcing (on $2^{<\omega}$). Then

$$J(\mathbb{P}) := \{X \subseteq 2^\omega : \forall p \in \mathbb{P} \ \exists q \in \mathbb{P} \text{ with } q \leq p \text{ and } [q] \cap X = \emptyset\}$$

denotes the null ideal associated to $\mathbb{P}$. (Analog., for tree forcings on $\omega^{<\omega}$.)

Examples:

- $s^0 := J(\mathbb{S})$ be the Marczewski ideal (or ideal of Marczewski null sets),
- $\ell^0 := J(\mathbb{L})$ be the ideal of Laver null sets,
- $m^0 := J(\mathbb{M})$ be the ideal of Miller null sets.

(all these ideals are σ-ideals – due to fusion)

Main Theorem (Spinas-W.)

Assume CH. Let $\langle(\mathbb{P}_\alpha, \dot{\mathbb{Q}}_\alpha) : \alpha < \omega_2\rangle$ be the countable support iteration with $\Vdash_{\mathbb{P}_\alpha} \dot{\mathbb{Q}}_\alpha = \dot{\mathbb{A}}$ for each $\alpha < \omega_2$.
Then in the final model $V^{\mathbb{P}_{\omega_2}}$ we have

$$\omega_1 = \mathrm{cov}(\mathcal{N}) = \mathrm{cov}(\mathcal{M}) = \mathfrak{h} < \mathrm{add}(s^0) = \mathfrak{s} = \mathfrak{b} = \mathfrak{c} = \omega_2.$$

Main Theorem (Spinas-W.)

Assume CH. Let $\langle(\mathbb{P}_\alpha, \dot{\mathbb{Q}}_\alpha) : \alpha < \omega_2\rangle$ be the countable support iteration with $\Vdash_{\mathbb{P}_\alpha} \dot{\mathbb{Q}}_\alpha = \dot{\mathbb{A}}$ for each $\alpha < \omega_2$.
Then in the final model $V^{\mathbb{P}_{\omega_2}}$ we have

$$\omega_1 = \mathrm{cov}(\mathcal{N}) = \mathrm{cov}(\mathcal{M}) = \mathfrak{h} < \mathrm{add}(s^0) = \mathfrak{s} = \mathfrak{b} = \mathfrak{c} = \omega_2.$$

Theorem (Goldstern-Repický-Shelah-Spinas)

1. If $\mathfrak{b} = \mathfrak{c}$, then $\mathrm{add}(\ell^0) \leq \mathfrak{h}$.
2. If $\mathfrak{d} = \mathfrak{c}$, then $\mathrm{add}(m^0) \leq \mathfrak{h}$.

Corollary

1. $CON(\mathrm{add}(\ell^0) < \mathrm{add}(s^0))$
2. $CON(\mathrm{add}(m^0) < \mathrm{add}(s^0))$

Corollary (from previous slide)

1. $CON(\mathrm{add}(\ell^0) < \mathrm{add}(s^0))$
2. $CON(\mathrm{add}(m^0) < \mathrm{add}(s^0))$

What about the cofinalities of these ideals?

Theorem (Fremlin; Judah-Miller-Shelah)

$\mathrm{cof}(s^0) > \mathfrak{c}$

Theorem (Brendle-Khomskii-W.)

$\mathrm{cof}(\ell^0) > \mathfrak{c}$ and $\mathrm{cof}(m^0) > \mathfrak{c}$

Corollary (from previous slide)

1. $CON(\mathrm{add}(\ell^0) < \mathrm{add}(s^0))$
2. $CON(\mathrm{add}(m^0) < \mathrm{add}(s^0))$

Theorem (Shelah-Spinas)

Let $\mathbb{Q}_0$, $\mathbb{Q}_1$ be forcings with $\mathbb{Q}_0, \mathbb{Q}_1 \in \{\mathbb{S}, \mathbb{L}, \mathbb{M}, \mathit{Silver}, \mathit{Mathias}\}$ such that

$$\omega_1 = \mathrm{add}(J(\mathbb{Q}_0)) < \mathrm{add}(J(\mathbb{Q}_1)) = \omega_2 \text{ is consistent,}$$

then

$$\mathrm{cof}(J(\mathbb{Q}_1)) < \mathrm{cof}(J(\mathbb{Q}_0)) = 2^{\mathfrak{c}} \text{ is consistent.}$$

Corollary (from previous slide)

1. $CON(\mathrm{add}(\ell^0) < \mathrm{add}(s^0))$
2. $CON(\mathrm{add}(m^0) < \mathrm{add}(s^0))$

Theorem (Shelah-Spinas)

Let $\mathbb{Q}_0$, $\mathbb{Q}_1$ be forcings with $\mathbb{Q}_0, \mathbb{Q}_1 \in \{\mathbb{S}, \mathbb{L}, \mathbb{M}, \mathit{Silver}, \mathit{Mathias}\}$ such that

$$\omega_1 = \mathrm{add}(J(\mathbb{Q}_0)) < \mathrm{add}(J(\mathbb{Q}_1)) = \omega_2 \text{ is consistent,}$$

then

$$\mathrm{cof}(J(\mathbb{Q}_1)) < \mathrm{cof}(J(\mathbb{Q}_0)) = 2^{\mathfrak{c}} \text{ is consistent.}$$

Corollary (in fact, the incentive for the Main Theorem)

1. $CON(\mathrm{cof}(s^0) < \mathrm{cof}(\ell^0))$
2. $CON(\mathrm{cof}(s^0) < \mathrm{cof}(m^0))$

Main Theorem (Spinas-W.)

Assume CH. Let $\langle(\mathbb{P}_\alpha, \dot{\mathbb{Q}}_\alpha) : \alpha < \omega_2\rangle$ be the countable support iteration with $\Vdash_{\mathbb{P}_\alpha} \dot{\mathbb{Q}}_\alpha = \dot{\mathbb{A}}$ for each $\alpha < \omega_2$.
Then in the final model $V^{\mathbb{P}_{\omega_2}}$ we have

$$\omega_1 = \mathrm{cov}(\mathcal{N}) = \mathrm{cov}(\mathcal{M}) = \mathfrak{h} < \mathrm{add}(s^0) = \mathfrak{s} = \mathfrak{b} = \mathfrak{c} = \omega_2.$$

Main Theorem (Spinas-W.)

Assume CH. Let $\langle (\mathbb{P}_\alpha, \dot{\mathbb{Q}}_\alpha) : \alpha < \omega_2 \rangle$ be the countable support iteration with $\Vdash_{\mathbb{P}_\alpha} \dot{\mathbb{Q}}_\alpha = \dot{\mathbb{A}}$ for each $\alpha < \omega_2$.
Then in the final model $V^{\mathbb{P}_{\omega_2}}$ we have

$$\omega_1 = \mathrm{cov}(\mathcal{N}) = \mathrm{cov}(\mathcal{M}) = \mathfrak{h} < \mathrm{add}(s^0) = \mathfrak{s} = \mathfrak{b} = \mathfrak{c} = \omega_2.$$

We only know of two other models in which $\mathfrak{h} < \mathfrak{s} = \mathfrak{b}$ holds true:

1. Shelah's model for $\omega_1 = \mathfrak{h} < \mathfrak{s} = \mathfrak{b} = \mathrm{cov}(\mathcal{M}) = \mathfrak{c} = \omega_2$
2. Jossen and Spinas' model for $\omega_1 = \mathrm{cov}(\mathcal{M}) = \mathfrak{h} < \mathfrak{s} = \mathfrak{b} = \mathfrak{c} = \omega_2$

$$\omega_1 = \mathrm{cov}(\mathcal{N}) = \mathrm{cov}(\mathcal{M}) = \mathfrak{h} < \mathrm{add}(s^0) = \mathfrak{s} = \mathfrak{b} = \mathfrak{c} = \omega_2.$$

For $f, g \in \omega^\omega$, we write $f \leq^* g$ if $f(n) \leq g(n)$ for all but finitely many n.

Definition ((un)bounding number)

$$\mathfrak{b} := \min\{|F| : F \subseteq \omega^\omega \ \wedge \ \neg \exists g \in \omega^\omega \ (f \leq^* g \text{ for each } f \in F)\}$$

Given $a, b \in [\omega]^\omega$, we say that a *splits* b if both $b \cap a$ and $b \setminus a$ are infinite.

Definition (splitting number(s))

$$\mathfrak{s} := \min\{|F| : F \subseteq [\omega]^\omega \ \wedge \ \forall b \in [\omega]^\omega \ \exists a \in F \ (a \text{ splits } b)\}$$

$$\mathfrak{s}_\sigma := \min\{|F| : F \subseteq [\omega]^\omega \wedge \forall \langle b_n \rangle_{n \in \omega} \subseteq [\omega]^\omega \ \exists a \in F \ \forall n \in \omega \ (a \text{ splits } b_n)\}$$

Clearly, $\mathfrak{s} \leq \mathfrak{s}_\sigma$.

Question (old open question)

Is it consistent with ZFC that $\mathfrak{s} < \mathfrak{s}_\sigma$?

Theorem (Hein-Spinas; for regular $\mathfrak{c}$: Simon, Judah-Miller-Shelah)

$\mathrm{add}(s^0) \leq \mathfrak{b}$.

Let

$$\mathfrak{a}(\mathbb{S}) := \min\{|A| : |A| > \aleph_0 \ \wedge \ A \subseteq \mathbb{S} \text{ is a maximal antichain}\}.$$

Clearly, $\aleph_1 \leq \mathfrak{a}(\mathbb{S}) \leq \mathfrak{c}$.

Theorem (Hein-Spinas)

$\mathfrak{d} \leq \mathfrak{a}(\mathbb{S})$.

Theorem (Hein-Spinas; for regular $\mathfrak{c}$: Simon,Judah-Miller-Shelah)

$\mathrm{add}(s^0) \leq \mathfrak{b}$.

Theorem (Hein-Spinas)

$\mathfrak{d} \leq \mathfrak{a}(\mathbb{S})$.

Corollary

$\mathrm{add}(s^0) \leq \mathfrak{a}(\mathbb{S})$.

Theorem (Spinas-W.)

Assume $\mathfrak{a}(\mathbb{S}) = \mathfrak{c}$. Then $\mathrm{add}(s^0) \leq \mathfrak{s}_\sigma$.

Corollary

$\mathrm{add}(s^0) = \mathfrak{c}$ implies $\mathfrak{s}_\sigma = \mathfrak{c}$.

Corollary (from previous slide)

$\mathrm{add}(s^0) = \mathfrak{c}$ implies $\mathfrak{s}_\sigma = \mathfrak{c}$.

Question

Does $\mathrm{add}(s^0) = \mathfrak{c}$ imply $\mathfrak{s} = \mathfrak{c}$?

Theorem (Spinas-W.; from previous slide)

Assume $\mathfrak{a}(\mathbb{S}) = \mathfrak{c}$. Then $\mathrm{add}(s^0) \leq \mathfrak{s}_\sigma$.

Theorem (Spinas-W.)

Assume $\mathfrak{a}(\mathbb{S}) = \mathfrak{c}$ and $\mathrm{add}(s^0) \leq \mathrm{cov}(\mathcal{M})$. Then $\mathrm{add}(s^0) \leq \mathfrak{s}$.

Corollary

$\mathrm{add}(s^0) = \mathrm{cov}(\mathcal{M}) = \mathfrak{c}$ implies $\mathfrak{s} = \mathfrak{c}$.

Given $p \in \mathbb{S}$, let $s_p = \langle s_p(i) : i < \omega \rangle$ be the canonical enumeration of the splitting nodes of p.

Definition (increasing Sacks amoeba, by Louveau-Shelah-Veličković)

Elements of $\mathbb{A}$ are pairs $\boldsymbol{p} = (p, n)$ such that

- $p \in \mathbb{S}$,
- $n < \omega$ (we call $n =: dp(\boldsymbol{p})$ the depth of $\boldsymbol{p}$),
- p is increasing (meaning that $\langle |s_p(i)| : i < \omega \rangle$ is strictly increasing).

Given $\boldsymbol{p} = (p, n^p), \boldsymbol{q} = (q, n^q) \in \mathbb{A}$, we let $\boldsymbol{q} \leqslant \boldsymbol{p}$ if

- $q \leq p$,
- $n^p \leq n^q$,
- $s_q(i) = s_p(i)$ for each $i \leq n^p$.

We say $\boldsymbol{q} \leqslant^0 \boldsymbol{p}$ ($\boldsymbol{q}$ is a pure extension of $\boldsymbol{p}$) if $\boldsymbol{q} \leqslant \boldsymbol{p} \wedge n^q = n^p$.

Moreover, we have stronger relations $\leqslant^m$ for constructing fusion sequences.

Theorem (Pure decision)

Suppose $N \in \omega$, $\boldsymbol{p} \in \mathbb{A}$, and $\boldsymbol{p} \Vdash \dot{\mu} \in N$.
Then there is $\boldsymbol{q} \leqslant^0 \boldsymbol{p}$ and $i \in N$ such that $\boldsymbol{q} \Vdash \dot{\mu} = i$.

Moreover: for each $m \in \omega$, there are $F \subseteq N$ with $|F| \leq 3^m$ and $\boldsymbol{q} \leqslant^m \boldsymbol{p}$ such that $\boldsymbol{q} \Vdash \dot{\mu} \in F$.

Theorem

$\mathbb{A}$ has the Laver property.

In particular, no Cohen reals and no random reals are added by $\mathbb{A}$, so

$$\mathrm{cov}(\mathcal{N}) = \mathrm{cov}(\mathcal{M}) = \omega_1$$

in the final model $V^{\mathbb{P}_{\omega_2}}$ of our Main Theorem.

$\mathbb{A}$ increases $\mathrm{add}(s^0)$ (actually the purpose of every Sacks amoeba):

Theorem (well-known)

Assume CH. Let $\langle(\mathbb{P}_\alpha, \dot{\mathbb{Q}}_\alpha) : \alpha < \omega_2\rangle$ be the countable support iteration with $\Vdash_{\mathbb{P}_\alpha} \dot{\mathbb{Q}}_\alpha = \dot{\mathbb{A}}$ for each $\alpha < \omega_2$.
Then in the final model $V^{\mathbb{P}_{\omega_2}}$ we have

$$\mathrm{add}(s^0) = \omega_2.$$

Several ways to to read off a dominating real d from the generic, e.g.

$$d(n) := |s_{p_G}(n)|,$$

where $s_{p_G} = \langle s_{p_G}(i) : i < \omega \rangle$ is the canonical enumeration of the splitting nodes of the generic tree p_G.

Theorem (easy)

$\mathbb{A}$ adds a dominating real. More specifically:

$$\Vdash \dot{d} \text{ is a dominating real.}$$

Hence

$$\mathfrak{b} = \omega_2$$

in the final model $V^{\mathbb{P}_{\omega_2}}$ of our Main Theorem.

Let $x \in [p_G]$ be the leftmost (or any) branch through the generic tree p_G; then

$$\mathrm{split}(p_G) \cap \{x \restriction n : n \in \omega\}$$

is not split by any set $a \in [2^{<\omega}]^\omega$ from the ground model. So:

Theorem (quite easy)

$\mathbb{A}$ adds a real which is not split by any ground model real.

Hence

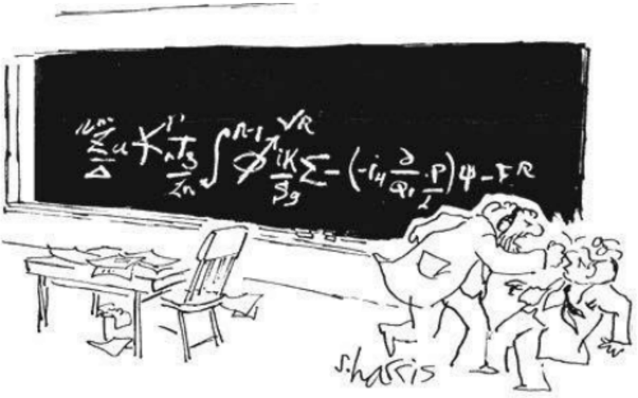
$$\mathfrak{s} = \omega_2$$

in the final model $V^{\mathbb{P}_{\omega_2}}$ of our Main Theorem.

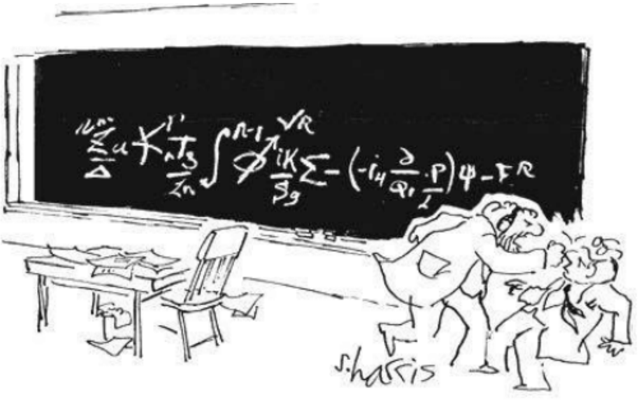
So why is

$$\mathfrak{h} = \omega_1$$

in the final model $V^{\mathbb{P}_{\omega_2}}$ of our Main Theorem?



"You want proof? I'll give you proof!"



"You want proof? I'll give you proof!"

http://www.math.uni-kiel.de/logik/de/spinas/a-sacks-amoeba-preserving-distributivity-of-p-omega-fin-with-w-wohofsky

Let $x, y \in [\omega]^\omega$.

- x is almost contained in y ($x \subseteq^* y$) if $x \setminus y$ is finite
- x and y are almost disjoint if $x \cap y$ is finite

Definition

A family $\mathcal{A} \subseteq [\omega]^\omega$ is an almost disjoint (ad) if

$$x \text{ and } y \text{ are almost disjoint for each } x, y \in \mathcal{A} \text{ with } x \neq y.$$

$\mathcal{A}$ is maximal almost disjoint (mad) family if it is a *maximal* ad family; in other words: and for each $z \in [\omega]^\omega$ there is an $x \in \mathcal{A}$ with $|z \cap x| = \aleph_0$.

Definition

$\mathcal{A}_1$ refines $\mathcal{A}_0$ if for each $x \in \mathcal{A}_1$ there is $y \in \mathcal{A}_0$ with $x \subseteq^* y$.

$\langle \mathcal{A}_\nu : \nu < \omega_1 \rangle$ is a matrix (of mad families) if $\mathcal{A}_\nu$ is mad for each $\nu < \omega_1$, and $\mathcal{A}_{\nu_1}$ refines $\mathcal{A}_{\nu_0}$ for any $\nu_0 \leq \nu_1 < \omega_1$.

Definition

We say that $a \in [\omega]^\omega$ intersects a matrix $\langle \mathcal{A}_\nu : \nu < \omega_1 \rangle$ if

$$\text{for each } \nu < \omega_1 \text{ there is a } y \in \mathcal{A}_\nu \text{ with } a \subseteq^* y.$$

A matrix is intersected if there is a real $a \in [\omega]^\omega$ intersecting it.

The following is more or less by definition:

Fact

$\mathfrak{h} = \omega_1$ if and only if there exists a matrix $\langle \mathcal{A}_\nu : \nu < \omega_1 \rangle$ of mad families which is not intersected.

Theorem (main part of our main theorem)

Assume CH. Let $\langle(\mathbb{P}_\alpha, \dot{\mathbb{Q}}_\alpha) : \alpha < \omega_2\rangle$ be the countable support iteration with $\Vdash_{\mathbb{P}_\alpha} \dot{\mathbb{Q}}_\alpha = \dot{\mathbb{A}}$ for each $\alpha < \omega_2$.

Then in the final model $V^{\mathbb{P}_{\omega_2}}$ we have a matrix which is not intersected, hence

$$\mathfrak{h} = \omega_1.$$

Proof.

- define the notion of a well-prepared matrix
- show that a well-prepared matrix is not intersected (never: not after forcing with $\mathbb{A}$, nor later – using Laver property)
- construct a well-prepared matrix in the ground model
- $\mathbb{A}$ adds dominating reals, so madness is destroyed all the time
- so we have to (cofinally) extend our matrix along the iteration □

Theorem

In V, let $\langle \mathcal{A}_\nu : \nu < \omega_1 \rangle$ be a well-prepared matrix.
Then

$$V^{\mathbb{A}} \models \text{ the matrix } \langle \mathcal{A}_\nu : \nu < \omega_1 \rangle \text{ is not intersected},$$

where $V^{\mathbb{A}}$ is the extension by a single Sacks amoeba forcing $\mathbb{A}$.

Moreover, the same holds true in any further Laver property extension:

$$V^{\mathbb{A} * \dot{\mathbb{R}}} \models \text{ the matrix } \langle \mathcal{A}_\nu : \nu < \omega_1 \rangle \text{ is not intersected},$$

where $\dot{\mathbb{R}}$ is forced to have the Laver property.

Theorem

In V, let $\langle \mathcal{A}_\nu : \nu < \omega_1 \rangle$ be a well-prepared matrix.
Then

$$V^{\mathbb{A}} \models \text{ the matrix } \langle \mathcal{A}_\nu : \nu < \omega_1 \rangle \text{ is not intersected},$$

where $V^{\mathbb{A}}$ is the extension by a single Sacks amoeba forcing $\mathbb{A}$.

Theorem

In V, let $\langle \mathcal{A}_\nu : \nu < \omega_1 \rangle$ be a well-prepared matrix.
Then

$$V^{\mathbb{A}} \models \text{ the matrix } \langle \mathcal{A}_\nu : \nu < \omega_1 \rangle \text{ is not intersected},$$

where $V^{\mathbb{A}}$ is the extension by a single Sacks amoeba forcing $\mathbb{A}$.

Definition

We call a matrix $\langle \mathcal{A}_\nu : \nu < \omega_1 \rangle$ a well-prepared matrix if

for each labelled condition $(\boldsymbol{q}, \lambda)$ there is $\boldsymbol{q}_0 \leqslant^0 \boldsymbol{q}$ and $\gamma < \omega_1$ such that the following holds:

$$\text{there is no } \boldsymbol{q}' \leqslant^0 \boldsymbol{q}_0 \text{ with } K(\boldsymbol{q}') \text{ “deciding” } \langle \mathcal{A}_\nu : \nu < \gamma \rangle.$$

Let $Ext(\boldsymbol{q})$ be the set of maximal elements of the set

$$\{\boldsymbol{r} \leqslant \boldsymbol{q} : dp(\boldsymbol{r}) = dp(\boldsymbol{q}) + 1\}.$$

For $\boldsymbol{r} = (r, n^r) \in \mathbb{A}$, let

$$\sigma(\boldsymbol{r}) := s_r(n^r),$$

i.e., the last splitting node of r which still belongs to the "fixed" part of $\boldsymbol{r}$.

Definition

Let $\boldsymbol{q} \in \mathbb{A}$. We say that $\lambda : Ext(\boldsymbol{q}) \to \omega$ is a labelling function for $\boldsymbol{q}$ if

$$\lambda(\boldsymbol{r}_0) \neq \lambda(\boldsymbol{r}_1) \text{ whenever } \boldsymbol{r}_0, \boldsymbol{r}_1 \in Ext(\boldsymbol{q}) \text{ with } \sigma(\boldsymbol{r}_0) \neq \sigma(\boldsymbol{r}_1).$$

We call $(\boldsymbol{q}, \lambda)$ a labelled condition.

Let $K(\boldsymbol{q}) := \{\lambda(\boldsymbol{r}) : \boldsymbol{r} \in Ext(\boldsymbol{q})\}$

Imagine $\dot{a}$ is an $\mathbb{A}$-name for a potential candidate for intersecting a matrix of mad families, i.e., an $\mathbb{A}$-name for an infinite subset of ω; the idea is to segment it by the interval partition

$$\{[d(n-1), d(n)) : n \in \omega\}$$

given by the dominating real d. We can assume without generality that

$$\Vdash |\dot{a} \cap [\dot{d}(n-1), \dot{d}(n))| \leq 1;$$

let $\dot{a}_n$ be such that

$$\Vdash \{\dot{a}_n\} = \dot{a} \cap [\dot{d}(n-1), \dot{d}(n))$$

in case $\dot{a} \cap [\dot{d}(n-1), \dot{d}(n))$ is non-empty, and

$$\Vdash \dot{a}_n = \mathrm{Empty}$$

otherwise.

We say that $f : Ext(\boldsymbol{p}) \to F$ is an open coloring if for $\boldsymbol{q} \in Ext(\boldsymbol{p})$, we have

$$\boldsymbol{q}' \leqslant^0 \boldsymbol{q} \implies f(\boldsymbol{q}') = f(\boldsymbol{q}).$$

BlackBox

Let $\boldsymbol{p} \in \mathbb{A}$, and $f : Ext(\boldsymbol{p}) \to F$ be open coloring with finitely many colors. Then there is $\boldsymbol{q} \leqslant^0 \boldsymbol{p}$ and $c \in F$ such that $f(\boldsymbol{r}) = c$ for each $\boldsymbol{r} \in Ext(\boldsymbol{q})$.

BlackBox

Let $\boldsymbol{p} \in \mathbb{A}$, and $f : Ext(\boldsymbol{p}) \to \omega$ be an open coloring.
Then there is $\boldsymbol{q} \leqslant^0 \boldsymbol{p}$ such that either (i) or (ii) holds:

1. there is an $\ell \in \omega$ such that $f(\boldsymbol{r}) = \ell$ for each $\boldsymbol{r} \in Ext(\boldsymbol{q})$,
2. $f(\boldsymbol{r}_0) \neq f(\boldsymbol{r}_1)$ whenever $\boldsymbol{r}_0, \boldsymbol{r}_1 \in Ext(\boldsymbol{q})$ with $\sigma(\boldsymbol{r}_0) \neq \sigma(\boldsymbol{r}_1)$.

Thank you for your attention and enjoy the Winter School...

Smrk 2019 (in fact, yesterday)

Thank you for your attention and enjoy the Winter School. . .

Smrk 2019 (in fact, yesterday)

Thank you for your attention and enjoy the Winter School...

Hejnice 2011

Thank you for your attention and enjoy the Winter School...

Hejnice 2011

Thank you for your attention and enjoy the Winter School...

Hejnice 2011